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Reception of Signals From A Distant Transmitting Half-Wave Dipole Excited by a Single-Cycle Sinusoid

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Characteristics of the time-dependent received voltage across the terminals of a thin dipole situated in the radiation zone of a thin transmitting dipole excited by a single-cycle sinusoidal voltage are investigated. In order to study this problem analytically, the zero-order approximate solution for the currents along the antennas in the frequency domain is used. Each of the four distinct incident electric field pulses induces four distinct (although overlapping) voltage pulses through the discontinuities of the receiving dipole antenna. When these two dipoles are mutually at the broadside of the other, the time duration of the incident electric field and the induced received voltage are lengthened by 1.5 cycles and up to 2 cycles respectively. It is observed that the time variation of the induced received voltage is smoother than the inducing (or incident) electric field. This shows that the receiving dipole behaves like an integrating circuit. Unlike a CW field, the spectrum of a short pulse incident electric field as well as the received voltage have peak values at frequencies higher than the carrier frequency f_0 of the exciting single-cycle sinusoid. Both the transmitting and the receiving dipoles attenuate lower frequencies of the spectra of the exciting single-cycle sinusoidal voltage.

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CONTENTS

RECEPTION OF SIGNALS FROM A DISTANT TRANSMITTING HALF-WAVE DIPOLE EXCITED BY A SINGLE-CYCLE SINUSOID

1. INTRODUCTION

In a previous work [1] we studied the behavior of the radiated electric field from a thin half-wave dipole in free space, excited by a single-cycle sinusoidal voltage with angular carrier frequency ω_0. This was investigated analytically by using an approximation, which consists of retaining only the zero-order solution for the current along the antenna in the frequency domain. This model of a transmitting dipole and its associated input impedance, which are based on zero-order approximation of the current without incorporating any radiation damping, was also considered by Franceschetti and Papas [2]. This model appears to be adequate as long as the observation point is far away from a thin transmitting dipole. It was shown in [1] that for a suitably matched dipole, the time-dependent radiated electric field was made up of five overlapping pulses indicating that radiation took place only from the discontinuities of the dipole antenna. In addition, it was found that both the current and the radiated electric field were extended in time due to the reflection of the current from the end points of the dipole. The spectral bandwidth of the radiated electric field was narrower than that of the exciting voltage pulse. On observing such characteristics of the radiation of a pulse form a transmitting dipole antenna, a question then naturally arises, "What will be the behavior of the voltage received by a similar receiving dipole, upon which the above-mentioned radiated field is incident?" The present work is devolted to answer that question using similar assumptions and approximations made in Ref. [1].

Figure 1 shows the geometry of the problem under study. The transmitting dipole AOB of length 2h, oriented along the z-axis of a coordinate system (x, y, z), is excited at its center O (which is also the origin of this coordinate system) by a single-cycle sinusoidal voltage $V_0 \sin \omega_0 t$ of duration T. A receiving dipole $A_r PB_r$ of length $2l$ is placed along the z_r -axis of another rectangular

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coordinate system $(x_r,\ y_r, z_r)$. the center of the receiving dipole $P\ (r_0,\ \theta_0,\ \phi_0)$, which is the origin of the coordinate system $(x_r,\ y_r,\ z_r)$, is in the far field of the transmitting dipole AOB with $|OP| = r_0$. The orientation of the receiving dipole can also be specified by two angles χ and ψ, where χ is the angle between PA_r and the z-axis, and ψ is the angle between the x-axis and the projection of PA_r in the $x-y$ plane. If one wishes, one may introduce the Eulerian angles to relate the two coordinate systems $(x,\ y,\ z)$ and $(x_r,\ y_r,\ z_r)$.

The problem is at first formulated in the frequency domain subject to the approximations mentioned above. To obtain the time dependent received voltage, the inverse Laplace transform is used with $s = i\omega$, where s and ω are the Laplace and the Fourier transform variables, respectively. This transform method is based on the assumption that the computed frequency response of the system is valid for all frequencies. However, this is not true always, especially when approximations are used in obtaining the frequency response. Therefore, the time domain result obtained in this way is not exact. Nevertheless, the present result which can be interpreted readily exhibits the dominant mechanisms of radiation and reception by a dipole.

It is found that each incident signal, which is radiated by the transmitting dipole, induces five signals at the receiving input terminals via five different paths. Since two of these signals look alike, because they have the same phase, only four distinct overlapping signals are induced at the center terminals of the receiving dipole antenna. It may be recalled [1] that the transmitted electric field also consists of five overlapping signals, two of which have the same phase, and thus showing four distinct radiated pulses. These phenomena show again that the mechanism of both the radiation and reception of electromagnetic pulses by dipoles is governed by the respective discontinuities. Among other characteristics of the received voltage, it is interesting to note that in order to receive the maximum energy from such radiated pulses, the receiver must be tuned at a frequency about 25% higher than the carrier frequency f_0 of the single-cycle exciting sinusoidal voltage. This is a sharp departure from the tuning requirement for the reception of CW signals. Detailed discussions of numerical results and theoretical analysis are presented in separate sections.

2. CHARACTERISTICS OF THE VOLTAGE RECEIVED BY A DIPOLE — DISCUSSION OF RESULTS

Figure 1 shows a receiving dipole A_rPB_r in the far field of a transmitting dipole antenna AOB excited by a single-cycle sinusoidal voltage $V_0 \sin \omega_0 t$ (where $\omega_0 = 2\pi f_0$). The voltage V_L is measured across a load impedance Z_L which connects the input terminals at the center of the receiving dipole. Since it is assumed that Z_L is real and independent of frequency, the behavior of the received antenna current and the received voltage is identical. It is found in Ref. [1], that at a distant point, P, in absence of the receiving antenna, the radiated field arrives from five different paths immediately after the transmitted antenna is excited at its feed point O. These paths are (Fig. 1) OP, OAP, OBP $OAOP$ and $OBOP$. The phases of the signals represented by paths $OAOP$ and $OBOP$ are the same. Therefore, an observer at P will see only four distinct overlapping signals. However, when a receiving dipole is placed with its center at P, each of the pulses (or signals) represented by the five paths mentioned above, will induce voltage pulses at the center of the receiving dipole in five different ways, since each arriving or incident signal does not necessarily take a single path for inducing voltage at the center of the receiving antenna respectively. For example, the incident signal represented by the path OAP will enter at P of the receiving dipole A_rPB_r via five different paths, namely, OAP, OAA_rP, OAB_rP, $OAPA_rP$ and $OAPB_rP$, before the corresponding voltages are induced at P. Since the paths PA_rP and PB_rP are equal, the phases of the induced signals represented by $OAPA_rP$ and $OAPB_rP$ are the same. Therefore, the signal which radiates from the point A of the transmitting dipole induces four distinct voltage pulses at the center of the receiving dipole. In this way one finds (Eqs. 16 and 24] that the induced voltage at the center of the receiving dipole consists of sixteen overlapping distinct signals. When the receiving dipole, instead of being oriented arbitrarily with respect to the transmitting dipole, is placed at the broadside of the transmitting antenna, the abovementioned sixteen distinct voltage signals reduce to nine.

In Fig. 2 the normalized time-dependent broadside radiated electric field $E_\theta(r_0, \theta_0 = \pi/2, t)/[V_0\zeta_0/(4\pi Z_{0r_0})]$, in the absence of or before being disturbed by the presence of the receiving dipole, is shown (dotted curve) as a function of a normalized time t^*f_0, where $t^* = t - r_0/c$ is the retarded time. Along with this radiated electric field, the normalized induced time-dependent load voltage $V_L(r_0, \theta_0, \theta_r, t)/[V_0/(2\beta_0 r_0\Omega]$ across the terminals of the receiving dipole, which is placed at the broadside ($\theta_0 = \pi/2 = \theta_r$) of the transmitting dipole, is also shown (solid curve) as a function of the normalized time t^*f_0. In this case the lengths of the transmitting and the receiving dipoles are half-wave at the carrier frequency f_0, i.e., $h = \ell = 0.25c/f_0$. Since this induced voltage and the incident (or radiated) electric field are normalized differently, their amplitudes should not be compared. Only their shapes as functions of the normalized time t^*f_0 may be of interest. Figure 2 shows that for a single-cycle sinusoidal exciting voltage ($\omega_0 T = 2\pi$), the radiated electric field before being received by the receiving antenna is extended in time to 1.5 cycles, whereas the duration of the induced load voltage across the terminals of the receiving antenna is increased to 2 cycles when $h = \ell$. The increase of the time duration of the radiated (or incident) signal is caused by the reflections of the induced current from the ends of the transmitting antenna. On the other hand, the additional time extension of the received voltage pulse is caused by the reflection of the incident electric field from the ends of the receiving antenna. It is interesting to note that the received voltage pulse has a smoother shape than that of the incident electric field. This observation suggests that during the process of reception of the incident field, the receiving antenna integrates the incoming signal in some sense. In other words, the receiving antenna acts like an integrating circuit. This is true and can be supported theoretically (see Eq. 24).

Figure 3 shows, using a given set of values of h and ℓ that when the length of the receiving dipole is shorter than the half-wave transmitting dipole ($\ell < h$), the duration of the received voltage will be between 1.5 to 2 cycles. It may then be inferred that the duration of the received voltage pulse may exceed 2-cycles, if the receiving antenna is made longer than $2h$. In Fig. 4 a normalized spectrum of the received voltage $|\beta_0 r_0 \hat{V}(\omega)|$ (see Eq. 27), as well as the spectrum of the exciting single-cycle sinusoidal voltage with different normalization, are presented as functions of the normalized frequency f/f_0. Because of different normalizations, their amplitudes should not be compared. This figure reveals another interesting characteristic of the received voltage pulse, when the transmitter is excited by a single-cycle sinusoidal voltage with carrier frequency $f_0 = \omega_0/2\pi$. The maximum energy in the received voltage is delivered at a frequency about 25% higher than f_0. Therefore, the receiver should be tuned at about 1.25 f_0. This phenomenon shows a sharp departure from the tuning requirement for the reception of CW signals. It is also shown in Ref. [1] that the peak value of the spectrum of the radiated electric field from a transmitting dipole occurs at a frequency $f > f_0$. However, if the number of cycles in the exciting sinusoid is increased, the value f of the peak of the spectrum of the radiated field, approaches f_0 gradually [1]. Consequently, it is expected that the spectrum of the received voltage will also show a similar behavior.

3. CONCLUSION

Characteristics of the received time-dependent voltage across the terminals at the center of a receiving dipole placed in the radiation zone of a transmitting dipole, excited by a single-cycle sinusoidal voltage are studied. The behavior of the corresponding radiated electric field from the transmitting dipole excited in the same manner was investigated previously [Ref. 1]. It is observed that the radiated electric field consists of four distinct overlapping pulses with an extended duration of 1.5 cycles. This increase in pulse length is caused by the reflections of current from from the end-points of the transmitting half-wave dipole. Each of these four radiated pulses then induces four distinct voltage pulses across the terminals of the receiving dipole. The process of reception of the incoming radiated field undergoes through reflections form the end points of the receiving antenna as the induced current travels along the latter. This mechanism causes additional increase of time duration so that the total duration is between 1.5 and 2 cycles of the received signal, depending on the length of the receiving dipole compared relative to that of the half-wave transmitter. It is also observed that both the processes of radiation from and reception by a dipole antenna take place via the respective discontinuities of the dipole concerned. The receiving antenna behaves like an integrating circuit so far as the incoming field is concerned. Since the spectra of both the radiated and received signals have peaks at a frequency higher than the carrier frequency f_0 of the exciting sinusoidal voltage, the receiving antenna should be tuned to a frequency higher (about 25%) than f_0. However, if the number of cycles in the exciting sinusoid is increased, the peak of the received spectrum approaches f_0. This phenomenon may be viewed as if the receiving dipole is filtering out lower frequencies further during the reception of short pulses.

APPENDIX

4. ANALYSIS

Consider a thin linear receiving dipole antenna of length 2ℓ situated in an arbitrary manner [Fig. 1] in the far field of a distant transmitting half-wave dipole of length $2h$, excited at its center by a single-cycle sinusoidal voltage $V_0 \sin \omega_0 t$. The transmitting dipole antenna lies along the z-axis of the coordinate system x, y, z; whereas the receiving dipole is oriented along the z_r-axis of the coordinate system x_r, y_r, z_r. The centers of these antennas separated by a very large distance r_0 are at the origins of the respective coordinate systems. It will be assumed that the direction cosines of the axes x_r,, y_r, z_r, with respect to the coordinate system x, y, z are known.

Our objective here is to calculate the open circuit induced voltage across the input terminals of the receiving dipole upon which the radiated field from the transmitting antenna is incident. From this result the voltage across a receiving load impedance Z_L can be determined easily. At first, the problem will be formulated in the frequency domain. Then the corresponding time domain result will be obtained by an application of the Fourier or Laplace transform. In this analysis we shall resort to the same kind of approximation, namely, the use of zero-order current distribution along the thin receiving dipole, as done for the transmitting dipole [1]. With this background in mind, let us assume that an electromagnetic field, represented by its electric vector $\vec{E}_i\,(r_0,\,\theta_0,\,\phi_0,\,\omega)$, in the frequency domain, is incident upon a receiving dipole, the center of which is located at $(r_0,\,\theta_0,\,\phi_0)$. The coordinates $(r_0,\,\theta_0,\,\phi_0)$ are assigned with respect to the transmitting dipole. Then an open circuit voltage $\hat{V}_{oc}(\theta_r,\,\phi_r;\,r_0,\,\theta_0,\,\phi_0;\,\omega)$, which we shall simply write as $\hat{V}_{oc}(\omega)$ for convenience, will be induced across the terminals of the receiving dipole. The spherical angles $(\theta_r,\,\phi_r)$, measured with respect to the receiving antenna, refer to the direction of the wave vector of the incident field as seen by an observer at the receiving antenna. It may be noted that the angles $(\theta_0,\,\phi_0)$ define the same direction as observed from the transmitting site. The induced open curcuit voltage $\hat{V}_{oc}(\omega)$ then can be expressed in the following way [3, 4].

$$\hat{V}_{oc}(\omega) = \vec{E}^{i}(r_0,\ \theta_0,\ \phi_0;\ \omega) \cdot \vec{h}_r(\theta_r,\ \phi_r;\ \omega), \tag{1}$$

where the vector effective height $\vec{h}_r(\theta_r,\ \phi_r,\ \omega)$ of the receiving antenna is defined by

$$\vec{h}_r(\theta_r,\ \phi_0,\ \omega) = \frac{[1 - \hat{r}_r\hat{r}_r]}{I_{in,r}} \int\int_{V_r}\int \vec{J}_r(\vec{r}_r,\ \omega)\exp\left\{i\,\frac{\omega}{c}\,(\hat{r}_r \cdot \vec{r}_r)\right\} dv_r. \tag{2}$$

In the above relation (2), the subscript r refers to the receiving antenna respectively. For example, $\hat{r}_r$ and $\vec{r}_r$ designate a unit vector in a radial direction and the position vector of a source element measured from the coordinate system of the receiving antenna. **1** is a unit dyadic. $\vec{J}_r(r_r,\ \omega)$ and $I_{in,r}$ are the induced current density and input current of the receiving antenna respectively. When the receiving dipole is along the z_r-axis, we have

$$\vec{h}_r(\theta_r,\ \phi_r,\ \omega) = \vec{h}_r(\theta_r,\ \omega) = -\ \hat{\theta}_r\,\frac{\sin\theta_r}{I_{in,r}}\int_{-\ell}^{\ell} I(z_r,\ \omega)\exp\left[i\,\frac{\omega}{c}\,z_r\cos\theta_r\right] dz_r, \tag{3}$$

where $\hat{\theta}_r$ is the unit vector in the θ_r-direction. For the zero-order approximation of the induced current, the above expression becomes [1]

$$\vec{h}_r(\theta_r,\ \omega) = -\hat{\theta}_r\,\frac{2\left[\cos\left(\frac{\omega\ell}{c}\cos\theta_r\right) - \cos\left(\frac{\omega\ell}{c}\right)\right]}{(\omega/c)\,\sin\theta_r \cdot \sin(\omega\ell/c)}. \tag{4}$$

In view of the reciprocity theorem, the vector effective height of the transmitting dipole antenna of length $2h$ and radiating in the θ_0-direction can be expressed in the following manner.

$$\vec{h}_t(\theta_0,\ \omega) = -\hat{\theta}_0\,\frac{2\left[\cos\left(\frac{\omega h}{c}\cos\theta_0\right) - \cos\left(\frac{\omega h}{c}\right)\right]}{(\omega/c)\ \sin\theta_0 \cdot \sin(\omega h/c)}. \tag{5}$$

where $\hat{\theta}_0$ is a unit vector in the θ_0-direction. Incidentally, it may be noted that the incident field $\vec{E}_i(r_0,\ \theta_0;\ \phi_0,\ \omega)$, which is the radiated electric field from the transmitting dipole antenna, can be expressed in terms of the transmitter effective height $\vec{h}_t(\theta_0,\omega)$ in the following way [1, 3].

$$\vec{E}_i\,(r_0, \theta_0, \phi_0;\,\omega) = \vec{E}_i\,(r_0,\,\theta_0;\,\omega) = \frac{-i\omega\zeta_0 e^{-i\frac{\omega}{c}r_0}}{4\pi r_0 c}\,I_{in,t}(\omega)\vec{h}_t(\theta_0,\,\omega), \tag{6}$$

where $I_{in,t}$ is the input current of the transmitting dipole and $\zeta_0 = \mu_0 c = \mu_0/\epsilon_0 =$ intrinsic impedance of the free space. The expression (6) was calculated previously [1], where the transmitting dipole was excited by a sinusoidal voltage of duration T. The generator of this voltage has an internal impedance $z_g = z_0 = \zeta_0\Omega/2\pi$, under which condition there is no reflection between the generator and the antenna terminals. Then the expression for $E^i(r_0,\,\theta_0,\,\omega)$ becomes [1]

$$\vec{E}^i(r_0,\,\theta_0,\,\omega) = \frac{-\hat{\theta}_0\zeta_0\hat{V}(\omega)}{2\pi r_0 Z_0\sin\theta_0}\left[\cos\left(\frac{\omega h}{c}\cos\theta_0\right) - \cos(\omega h/c)\right]\exp\left\{-\frac{i\omega}{c}(r_0+h)\right\}, \tag{7}$$

where

$$\hat{V}(\omega) = V_0\omega_0[1 - \exp(-i\omega T)F(i\omega,\,T)]/(\omega_0^2 - \omega^2), \tag{8}$$

$$F(i\omega,\,T) = (i\omega/\omega_0)\sin\omega_0 T + \cos\omega_0 T, \tag{9}$$

and V_0 is the amplitude of the sinusoidal voltage of duration T. The voltage across a load impedance Z_L connected across the input terminals of the receiving antenna is then given by

$$\hat{V}_L(\omega) = \frac{Z_L\hat{V}_{oc}(\omega)}{Z_r + Z_L} = \frac{Z_L\vec{E}^i(r_0,\,\theta_0,\omega)\cdot\vec{h}_r(\theta_r,\,\omega)}{Z_r + Z_L)} \tag{10}$$

where Z_r is input impedance of the receiving antenna, which will be the same, when this antenna is used as a transmitter. Therefore, we have, using the same assumptions and approximation made in Ref. [1],

$$Z_r = Z_t = -iZ_0\ \cot(\omega\ell/c). \tag{11}$$

Then taking $Z_L = Z_g = Z_0$, the expression (10) can be rewritten in the following way, where use has been made of equations (4) and (7)-(9) with $i\omega = s$.

$$\hat{V}_L(-is) = G(\beta_0 r_0, \theta_0, \theta_r) \cdot \left[4\omega_0^2 \frac{\{e^{-sT}F(s, T) - 1\}}{s(s^2 + \omega_0^2)} \right] \cdot [\cosh \left(\frac{sh}{c} \cos\theta_0 \right) - \cosh \left(\frac{sh}{c} \right)] \tag{12}$$

$$\cdot \left[\cosh \left(\frac{s\ell}{c} \cos\theta_r \right) - \cosh \left(\frac{s\ell}{c} \right) \right] \exp\{s(r_0 + h + \ell)/c\},$$

where $\beta_0 = \omega_0/c$ and s is the Laplace transform variable.

$$G(\beta_0, r_0, \theta_0, \theta_r) = \frac{(\hat{\theta}_0 \cdot \hat{\theta}_r) V_0}{2(\beta_0 r_0)\, \Omega \sin\theta_0 \sin\theta_r} \tag{13}$$

Then the time-dependent receiving load voltage is given by the following inverse Laplace transform of $\hat{V}_L(-is)$.

$$V_L(r_0, \theta_0, \theta_r, t^*) = \frac{1}{2\pi i} \int_\Gamma \hat{V}_L(-is) e^{st}\, ds, \tag{14}$$

where $t^* = t - r_0/c$ is the retarded time. It can easily be seen using the L'Hospital's rule that $s = 0$ is not a pole of $V_L(-is)$. However, it will be necessary to express the hyperbolic cosine functions in terms of exponentials, in which case for an individual term in (12) or (13), $s = 0$ will become a pole. Therefore, the contour Γ in (14) is chosen along the imaginary axis of the complex s-plane with indentations from the right side at the poles $s = 0$ and $s = \pm i\omega_0$. Finally, the contour is closed by a semi-circle in the left-half plane at infinity, so that the poles lie inside this closed contour. Then the time-dependent load voltage $V_L(r_0, \theta_0, \theta_r, t^*)$ can be expressed in the following form.

$$V_L(r_0, \theta_0, \theta_r, t^*) = G(\beta_0 r_0, \theta_0, \theta_r)\, S(\theta_0, \theta_r, t^*), \tag{15}$$

where $S(\theta_0, \theta_r, t^*)$ is the Laplace transform of $V_L(r_0, \theta_0, \theta_r, t^*)/\, G(\beta_0 r_0, \theta_0, \theta_r)$.

The time function $S(\theta_0, \theta_r, t^*)$ consists of twenty-five signals, each of which has time duration T and appears at different overlapping time intervals. Since some signals have the same phases, there are only sixteen distinct signals. Assuming that $T = nT_0$, where n is a positive integer and $T_0 = 1/f_0$ = a period of a single-cycle, the function $S(\theta_0, \theta_r, t^*)$ can be represented in terms of unit step functions $U(t_i)$ and $U(t_i - T)$, with $i = 1$ to 16.

$$
\begin{aligned}
S(\theta_0, \theta_r, t^*) = {} & [\cos \omega_0 t_1 - 1] [U(t_1) - U(t_1 - T)] \\
& + [\cos (\omega_0 t_2) - 1] [U(t_2) - U(t_2 - T)] \\
& - [\cos (\omega_0 t_3) - 1] [U(t_3) - U(t_3 - T)] - [\cos (\omega_0 t_4) - 1] [U(t_4) - U(t_4 - T)] \\
& - [\cos (\omega_0 t_5) - 1] [U(t_5) - U(t_5 - T)] + [\cos (\omega_0 t_6) - 1] [U(t_6) - U(t_6 - T)] \\
& - [\cos (\omega_0 t_7) - 1] [U(t_7) - U(t_7 - T)] - [\cos (\omega_0 t_8) - 1] [U(t_8) - U(t_8 - T)] \\
& - [\cos (\omega_0 t_9) - 1] [U(t_9) - U(t_9 - T)] - [\cos (\omega_0 t_{10}) - 1] [U(t_{10}) - U(t_{10} - T)] \\
& + [\cos (\omega_0 t_{11}) - 1] [U(t_{11}) - U(t_{11} - T)] - [\cos (\omega_0 t_{12}) - 1] [U(t_{12}) - U(t_{12} - T)] \\
& + [\cos (\omega_0 t_{13}) - 1] [U(t_{13}) - U(t_{13} - T)] - [\cos (\omega_0 t_{14}) - 1] [U(t_{14}) - U(t_{14} - T)] \\
& - [\cos (\omega_0 t_{15}) - 1] [U(t_{15}) - U(t_{15} - T)] + [\cos (\omega_0 t_{16}) - 1] [U(t_{16}) - U(t_{16} - T)]
\end{aligned}
\tag{16}
$$

Each of the quantities $t_1, t_2, \ldots, t_{16}$, defined below, represents the respective time taken by an individual signal to travel from the input terminals of the transmitting dipole to those of the receiving dipole via various discontinuities (input and end-points) of both antennas [Fig. 1].

$t_1 = t^* = t - r_0/c =$ retarded time associated with the travel time along the path OP, (17a)

(17b)

$t_2 = t_1 - 2\ell/c =$ retarded time associated with the travel time along each of the paths, OPA_rP and OPB_rP, for which the phases are the same. This shows that the second term of (16) is contributed by these two paths equally.

(17c)

$t_3 = t_1 - \ell(1 - \cos\theta_r)/c =$ retarded time associated with the travel time along the path OA_rP.

(17d)

$t_4 = t_1 - \ell(1 + \cos\theta_r)/c =$ retarded time associated with the travel time along the path OB_rP.

(18a)

$t_5 = t_1 - h(1 - \cos\theta_0)/c =$ retarded time associated with the time along the path OAP,

(18b)

$t_6 = t_1 - h(1 - \cos\theta_0)/c - \ell(1 - \cos\theta_r)/c =$ retarded time associated with the travel time along the path OAA_r

(18c)

$t_7 = t_1 - h(1 - \cos\theta_0)/c - \ell(1 + \cos\theta_r)/c =$ retarded time associated with the travel time along the path OAB_rP

(18d)

$t_8 = t_1 - h(1 - \cos\theta_0)/c - 2\ell/c =$ retarded time associated with the travel time along each of the paths $OAPA_rP$ and $OAPB_rP$. This means that the 8th term of (16) is contributed equally by these two paths.

(19a)

$t_9 = t_1 - h(1 + \cos\theta_0)/c =$ retarded time associated with the travel time along the path OBP,

(19b)

$t_{10} = t_1 - h(1 + \cos\theta_0)/c - \ell(1 + \cos\theta_r)/c =$ retarded time associated with the travel time along the path OBB_rP.

(19c)

$t_{11} = t_1 - h(1 + \cos\theta_0)/c - \ell(1 - \cos\theta_r)/c =$ retarded time associated with the travel time along the pain OBA_rP,

(19d)

$t_{12} = t_1 - h(1 + \cos\theta_0)/c - 2\ell/c =$ retarded time associated with the travel time along each of the paths, $OBPA_rP$ and $OBPB_rP$. Therefore, the 12^{th} term of (16) is contributed equally by these two paths.

(20a)

$t_{13} = t_1 - 2h/c =$ retarded time associated with the travel time along each of the paths, $OAOP$ and $OBOP$. This s. ws that the 13^{th} term of (16) is equally contributed by these two paths.

(20b)

$t_{14} = t_1 - 2h/c - \ell(1 - \cos\theta_r)/c =$ retarded time associated with the travel time along each of the paths $OAOA_rP$ and $OBPB_rP$. Therefore, the 14^{th} term of (16) is equally contributed by these two paths.

(20c)

$t_{15} = t_1 - 2h/c - \ell(1 + \cos\theta_r)/c =$ retarded time associated with the travel time along each of the paths, $OAOBP$ and $OBOB_rP$. Therefore, the 15^{th} term of (16) is equally contributed by these two paths.

(20d)

$t_{16} = t_1 - 2h/c - 2\ell/c =$ retarded time associated with the travel time along each of the paths, $OAOPA_rP$, $OAOPB_rP$, $OBOPA_rP$ and $OBOPB_rP$. Therefore, the last term of (16) is equally contributed by these four paths.

It is interesting to notice that in the absence of the receiving dipole, the four distinct signals which would have radiated from the transmitting dipole to a distant point P can be identified with the terms associated with the travel times t_1, t_5, t_9 and t_{13}, respectively. Each of these four signals then induces four distinct voltage pulses at the terminals of the receiving dipole. It should be understood that the signals associated with the travel times t_1, t_5, t_9 and t_{13}, repectively, also constitute a portion of the total induced voltage pulse when the receiving dipole is present.

When the receiving and the transmitting dipoles are at the broadside of each other, i.e., when $\theta_0 = \pi/2 = \theta_r$, one finds that $t_3 = t_4 = t_1 - \ell/c$, $t_5 = t_9 = t_1 - h/c$, $t_6 = t_7 = t_{10} = t_{11} = t_1 - h/c - \ell/c$, $t_8 = t_{12} = t_1 - h/c - 2\ell/c$ and $t_{14} = t_{15} = t_1 - 2h/c - \ell/c$. Therefore, the sixteen distinct signals given by Eq (16), reduce to nine distinct signals.

The result (16) is obtained from (14) via residue calculus. It can also be extracted from (14) by using some theorems associated with the Laplace transform. In particular the following relations are found to be helpful.

$$L^{-1}[\exp\{-st_i\} \cdot L(g(t)\}] = g(t - t_i) \cdot U(t - t_i) \tag{21a}$$

$$L^{-1}[\exp\{-s(t_i + T)\} \cdot L\{g(t + T)\}] = g(t - t_i) \cdot U(t - t_i - T) \tag{21b}$$

$$L[\{U(t - t_i) - U(t - t_i - T)\}] \cdot \int_0^{t - t_i} g(\tau) d\tau = \tag{21c}$$

$$(1/s) \cdot \int_{t_d}^{t_i + T} e^{-st}\, g(t - t_i)\, dt - (1/s) \cdot \exp\{-s(t_i + T)\} \cdot \int_0^T g(\tau)\, d\tau.$$

where the symbols L and L^{-1} stand for the Laplace transform operation and its inverse respectively.

$$\int_{t_i}^{t} g(\tau - t_i)\, [U(\tau - t_i) - U(\tau - t_i - T)]\, d\tau$$

$$= [U(t - t_i) - U(t - t_i - T)] \cdot \int_{t_i}^{t} g(\tau - t_i)\, d\tau, \tag{22}$$

$$\text{provided } \int_0^T g(t)\, dt = 0.$$

Using these relations, which can be established easily, one can then show from equation (14), that when the exciting voltage is

$$V(t) = V_s(t) \cdot [U(t) - U(t - T)], \tag{23}$$

where $V_s(t)$ stands for $g(t)$ in the relations (21a) to (22), then

$$V_L(r_0, \theta_0, \theta_r, t^*)/[\hat{\theta}_0 \cdot \hat{\theta}_r/\{2(r_0/c)\Omega \sin\theta_0 \sin\theta_r\}]$$

$$= -[U(t_1) - ((t - T)] \int_0^{t_1} V_s(\tau)\, d\tau - [U(t_2) - U(t_2 - T)]\int_0^{t_2} V_s(\tau)\, d\tau$$

$$+ [U(t_3) - U(t_3 - T)] \cdot \int_0^{t_3} V_s(\tau)\, d\tau + [U(t_4) - U(t_4 - T)] \cdot \int_0^{t_4} V_s(\tau) d\tau$$

$$+ [U(t_5) - U(t_5 - T)] \cdot \int_0^{t_5} V_s(\tau)\, d\tau - [U(t_6) - U(t_6 - T)] \cdot \int_0^{t_6} V_s(\tau) d\tau$$

$$- [U(t_7) - U(t_7 - T)] \cdot \int_0^{t_7} V_s(\tau)\, d\tau - [U(t_8) - U(t_8 - T)] \cdot \int_0^{t_8} V_s(\tau) d\tau$$

$$+ [U(t_9) - U(t_9 - T)] \cdot \int_0^{t_9} V_s(\tau)\, d\tau - [U(t_{10}) - U(t_{10} - T)] \cdot \int_0^{t_{10}} V_s(\tau) d\tau$$

$$- [U(t_{11}) - U(t_{11} - T)] \cdot \int_0^{t_{11}} V_s(\tau)\, d\tau + [U(t_{12}) + U(t_{12} - T)] \cdot \int_0^{t_{12}} V_s(\tau) d\tau$$

$$- [U(t_{13}) - U(t_{13} - T)] \cdot \int_0^{t_{13}} V_s(\tau)\, d\tau + [U(t_{14}) - U(t_{14} - T)] \cdot \int_0^{t_{14}} V_s(\tau) d\tau$$

$$+ [U(t_{15}) - U(t_{15} - T)] \cdot \int_0^{t_{15}} V_s(\tau)\, d\tau - [U(t_{16}) - U(t_{16} - T)] \cdot \int_0^{t_{16}} V_s(\tau) d\tau \tag{24}$$

The expression (24) shows that the receiving antenna is behaving like an integrating circuit. Similarly, replacing ω by $-is$ in (7) ana th.:n taking its inverse Laplace transform, the time-dependent radiated electric field can be expressed in the following manner [using 21a and 21b].

$$\hat{\theta}_0 \cdot \vec{E}^i(r_0, \theta_0, T)/[\zeta_0/(4Z_0\, \pi\, r_0 \sin\theta_0)]$$

$$= [U(t_1) - U(t_1 - T)] \cdot V_s(t_1) - [U(t_5) - U(t_5 - T)]\, V_5(t_5), \tag{25}$$

$$- [(U(t_s) - U(t_9 - T)] \cdot V_s(t_9) + [U(t_{13}) - U(t_{13} - T)] V_s(t_{13}).$$

The corresponding modified zero-order time-dependent antenna current on the transmitting dipole can also be represented using the same principle.

$$I_{OM}(z, t)/(2Z_0)$$

$$= [U(t - |z|/c - U(t - |z|/c - T)] V_s(t - |z|/c)$$

$$- [U\{t - (2h - |z|)/c\} - U\{(t - (2h - |z|)/c - T\}] V_s(t - (2h - |z|)/c). \qquad (26)$$

Observing now the similarities between (25) above and (12) of ref. [1] and (26) above and (13) of ref. [1], the results (25) and (26) can be interpreted in the same manner, i.e., discontinuities of an antenna play a dominant role in radiation mechanism.

Comparing the expression (7) and (12) with $s = i\omega$, it can be seen that the spectrum of the radiated field from the transmitting dipole is not the same as that of the induced load voltage of the receiving dipole. The reason for this phenomenon may be explained in the following way. Since antennas (the transmitting as well as the receiving dipoles) behave like frequency filters, the frequency spectrum of the incident field is expected to be filtered further by the receiving dipole. We have already observed [1] that when a single-cycle sinusoidal voltage excites a transmitting dipole, the spectrum of the radiated field is attenuated as well as filtered.

The expression for the spectrum of the received load voltage when $\theta_0 = \pi/2 = \theta_r$, i.e., when both the antennas are mutually at each other's broadside, reduces to the following form (with $V_0 = 1$).

$$|\beta_0 r_0 \hat{V}_L(\omega)| = \frac{T\omega_0}{\omega} (1 - \cos \omega h/c)(1 - \cos \omega \ell/c) \cdot$$

$$\cdot \left[\left\{ \frac{\sin\{T(\omega - \omega_0)\}}{T(\omega - \omega_0)} - \frac{\sin\{T(\omega + \omega_0)\}}{T(\omega + \omega_0)} \right\} - i \left\{ \frac{\sin^2\{T(\omega - \omega_0)/2\}}{T(\omega - \omega_0)/2} - \frac{\sin^2\{T(\omega + \omega_0)/2\}}{T(\omega + \omega_0)/2} \right\} \right] \qquad (27)$$

In (27) the quantity in the square bracket multiplied by T represents the spectrum of the exciting sinusoidal voltage of duration T.

5. ACKNOWLEDGMENT

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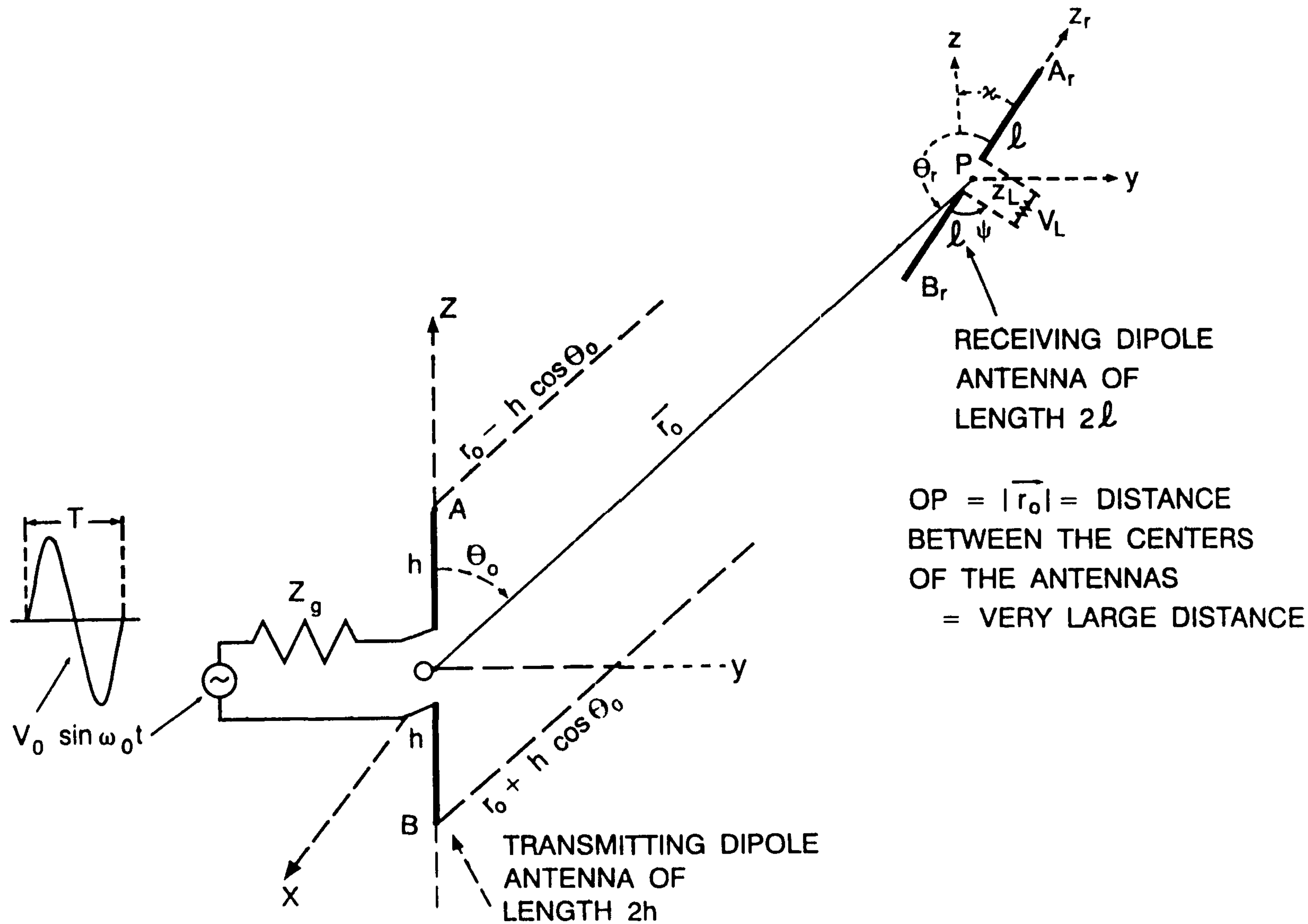

Fig. 1 — Transmitting and receiving Dipole Antennas (They may not be in the same plane)

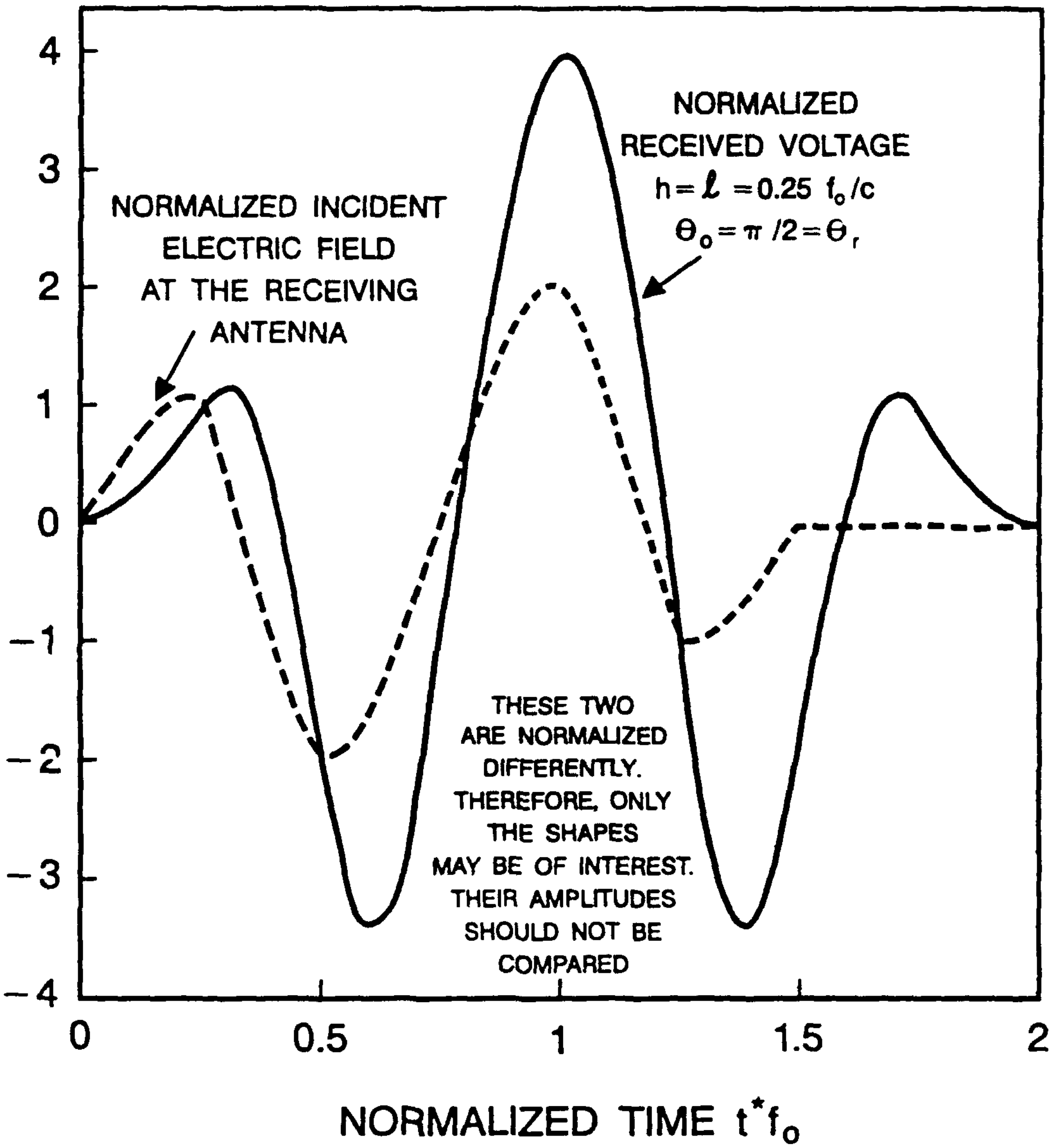

Fig. 2 — Normalized Radiated Electric Field and Received voltage when the Antennas are at Broadside of each other and $h = \ell$

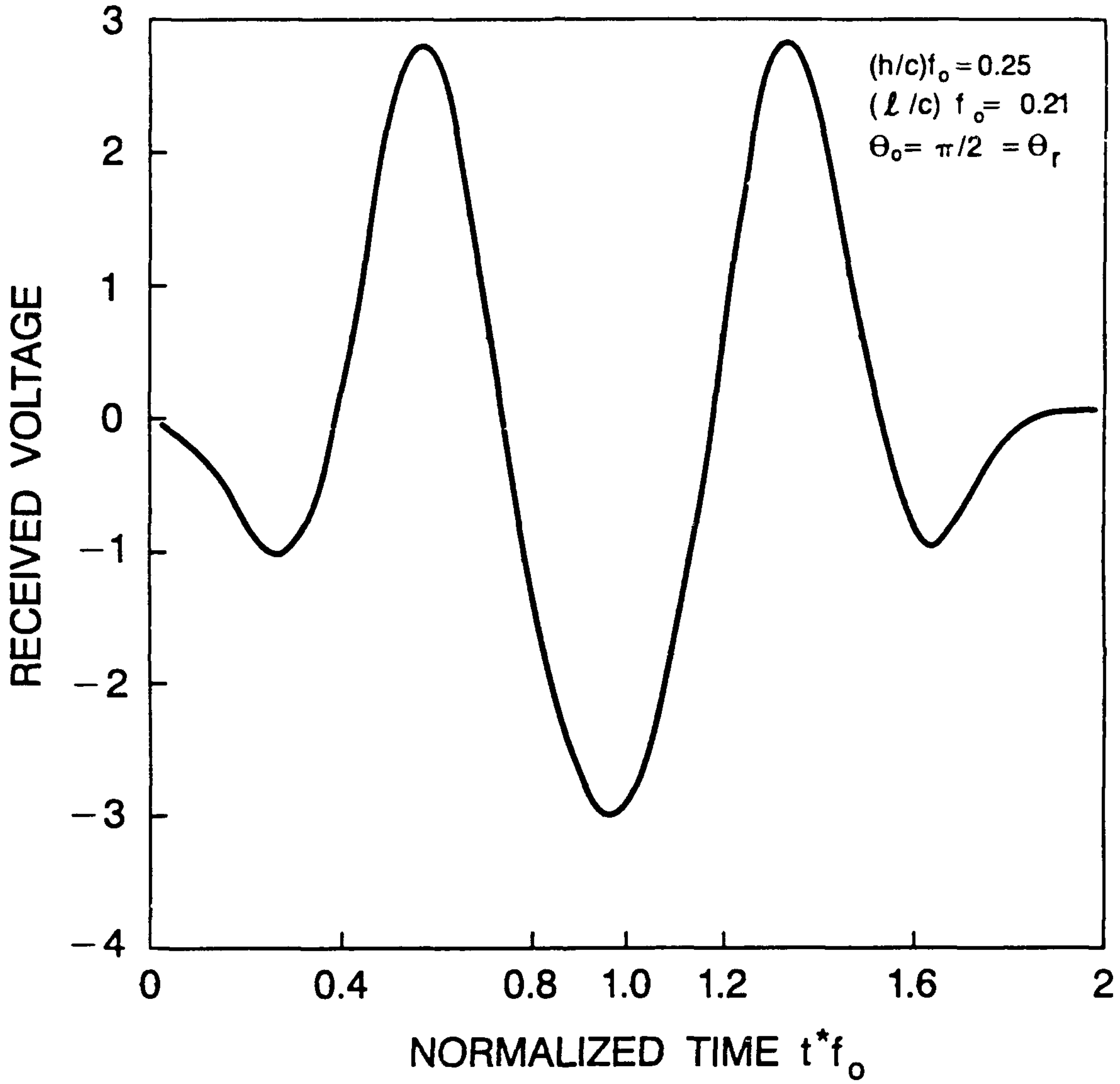

Fig. 3 — Normalized Received voltage when $\ell < h$ and the Antennas are at Broadside of each other

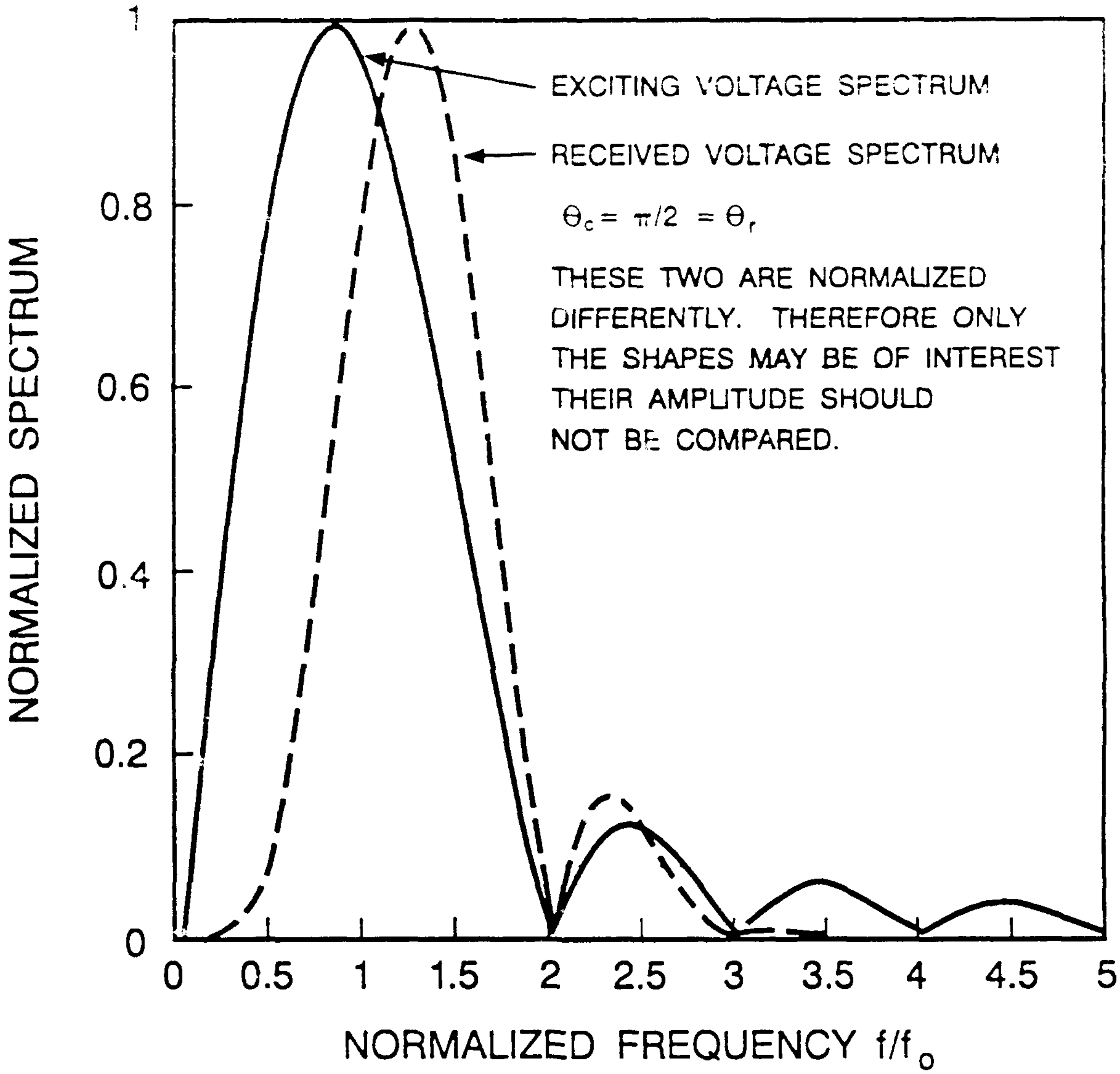

Fig. 4 — Normalized Exciting voltage and Received voltage spectra